De-Embedding Correction for Imperfect Absorbing Boundary Conditions in FDTD

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Abstract— **The finite differences in the time domain (FDTD) technique is a very powerful method for field evaluation in complex structures. In many cases knowledge of the field is unnecessary, because that of the scattering parameter suffices. We introduce a very simple, fast, and easy-to-implement method to correct the absorbing boundary condition (ABC) errors in the scattering parameter evaluation arising from any ABC, however imperfect: this method allows universal use of any simple ABC for any transmission media, therefore facilitating the development of general purpose codes. The technique is based on the evaluation of the reflection coefficient due to the absorbing boundary condition and a subsequent correction in the frequency domain.**

I. Introduction

The use of FDTD allows the full-wave analysis of many complex structures. ABC's produce some errors due to the nonideal truncations, and in recent years many efforts have been directed toward eliminating them. In the analysis and design of devices, knowledge of the scattering parameters is sufficient in order to describe the components, whereas field knowledge is often unnecessary.

Lin and Naishadham [1] introduced an easy technique for reducing the error in the evaluation of transmission coefficients due to the ABC by considering the truncations of the incident field structure and of the total field structure at identical distances. This approach eliminates the effect of only the first reflection in evaluating the transmission coefficient; it cannot correct the reflection coefficient and it fails when multiple reflections are introduced by poor ABC's.

We analyze the complete circuit model of the overall structure, including the incident line, the line containing discontinuities, the separation plane, and the ABC. We evaluate first the reflection coefficients of the absorbing planes due to the imperfect ABC by analysing a known element, i.e. a section of line. This computation uses a reduced structure and is carried out at the beginning as a "de-embedding" of the FDTD working geometry. The data are stored and recalled in the post-processing of the subsequent FDTD analysis where we use those reflection coefficients in order to correct the scattering parameter of the unknown element.

II. Method Description

Fig. 1 shows the circuit model of the FDTD analysis. The $\mathbf{E}$-matrix in Fig. 1(b) is the scattering matrix of the four-port simulating the separation of the reflected field from the total field. Fig. 1(a) shows the known procedure for obtaining separation: we use a uniform auxiliary line in order to reproduce the exact incident field distribution [this is shown in upper line in Fig. 1(a)]. At the separation plane P the electric or magnetic fields are added or subtracted in the lower line in order to excite the right part of the line while leaving only the reflected field in the left part. The separation plane does not introduce any phase shift. The signal incoming at port 1 is transmitted to port 2 and reproduced at port 4 so that $S_{21} = S_{41} = 1$; the signal incoming at port 2 is transmitted to port 1 so that $S_{12} = 1$; the signal incoming at port 3 is transmitted to port 4 and vice versa so that $S_{43} = S_{34} = 1$. No other coupling is present. The resulting $\mathbf{E}$-matrix assumes the form

$$\mathbf{E} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{vmatrix}.$$

It is noted that $\mathbf{E}$ is not unitary, since energy is not conserved by the above separation. The $\mathbf{S}$-matrix is either the scattering matrix of the known element used to find the normalized admittance of the terminations Y or of the unknown element to be analyzed. I, R, and T are the planes where we evaluate the fields for the purpose of scattering parameter evaluation. Excitation of the incident line is achieved by imposing the field at the beginning of the line and it is equivalent to an ideal voltage generator. We excite the line with a sinusoidally modulated pulse and produce the responses V_I, V_R and V_T by FDTD analysis. A fast fourier transform (FFT) translates those voltages into the frequency domain.

The circuit of Fig. 1(b) is easily solved in the frequency domain twice.

In the first run we evaluate the unknown Y elements by analyzing a known section of line. The normalized Y value can be obtained in three separate ways by considering $V_I/V_0, V_T/V_I$ or V_R/V_I. The first case, involving fields in the incident line only, does not yield actual results because the excitation fields may be unknown for many kinds of line. For example, in the microstrip case, the field distribution in the transverse plane varies with frequency so that excitations ought to be effected by means of an average distribution and the first line should be to quite long in order to match the field to the structure. The two other cases give similar, good results. As an example, by using the reflected fields, assuming $L_4 = L_1 = L_2 + L_3$ and normalized characteristic impedance

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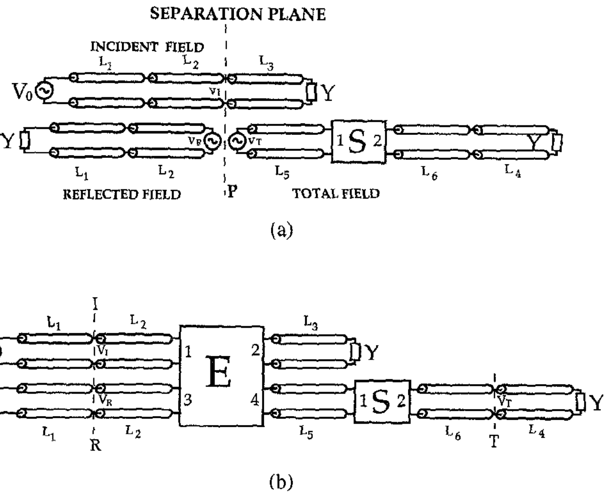

Fig. 1. (a) Standard circuit model for S-parameter evaluation by FDTD method. (b) The **E**-matrix effecting the separation between the reflected, total, and incident field. **S** is a scattering matrix, known or unknown.

of the lines, we obtain for a symmetric structure

$$a_r Y^2 + b_r Y + c_r = 0$$

where

$$a_r = j\left[\sin(\theta_T)\left(\frac{V_R}{V_I} - 1\right) + \frac{V_R}{V_I}\cos(\theta_T)\sin(2\theta_{L_4}) - 2\sin(\theta_T)\frac{V_R}{V_I}\sin^2(\theta_{L_4})\right]$$

$$b_r = -2\frac{V_R}{V_I}\sin(\theta_T)\sin(2\theta_{L_4}) + 2\cos(\theta_T)\frac{V_R}{V_I}(1 - 2\sin^2(\theta_{L_4}))$$

$$c_r = j\left[\sin(\theta_T)\left(\frac{V_R}{V_I} + 1\right) + \frac{V_R}{V_I}\cos(\theta_T)\sin(2\theta_{L_4}) - 2\sin(\theta_T)\frac{V_R}{V_I}\sin^2(\theta_{L_4})\right].$$

θ_T is the electric length from the R and T planes and θ_{L_4} is the electric length of the lines $L_4 = L_1 = L_2 + L_3$.

The above equation gives two solutions, only one being acceptable.

As a second step, we solve the circuit of Fig. 1(b) with the computed Y-admittances by evaluating and correcting the S-parameters of any unknown element.

III. Result

In order to test the method we analyze and correct a considerable residual reflection arising from terminating a waveguide by means of the first-order Mur's ABC [2]; the results are shown in Fig. 2. In the same figure we also show some results using a waveguide terminated by means of a modified second-order Mur's ABC [3]. The corresponding normalized admittances for the two previous conditions are shown in Fig. 3. Fig. 2 also shows the magnitude of the S_{11} parameter of two different lengths of waveguide. The troughs in the graphs are due to periodically vanishing fields along the longer lines. Correction improves the performance by about 50 dB in all cases.

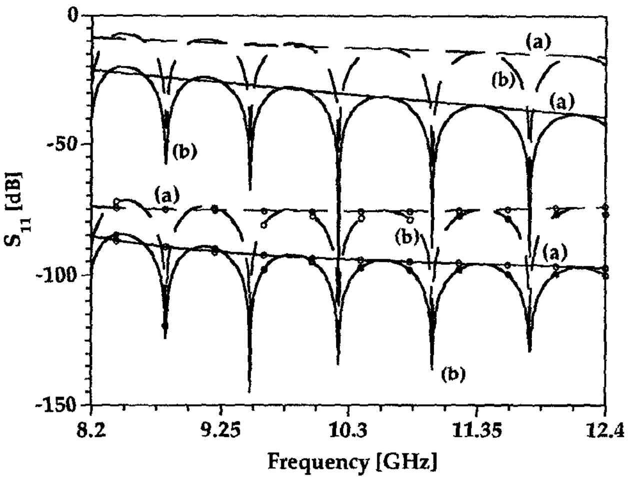

Fig. 2. Corrected (marked with "O") and uncorrected (unmarked) magnitude of S_{11} for two different lines: (a) length = 7.5 mm; (b) length = 153 mm; waveguide sides: 22.86 × 10.16 mm. The resonance peaks of the longer line are due to the vanishing of the electric field at the plane R. Continuous lines refer to modified second-order Mur's ABC. Dashed lines refer to first-order Mur's ABC.

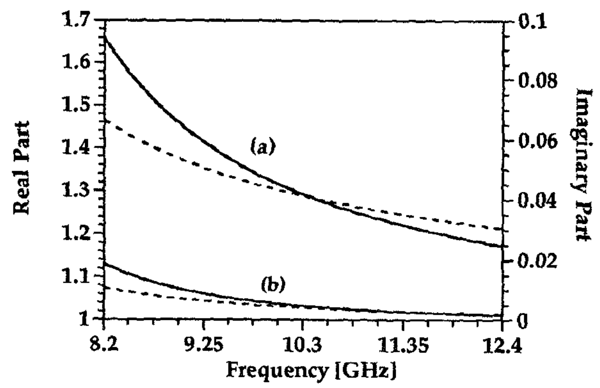

Fig. 3. Real (——) and imaginary (- - - -) parts of Y obtained by imposing the first (a) or the modified second-order Mur's ABC (b).

We then analyze a twin inductive post filter in a waveguide previously studied by means of a variational method and measured [4]. Fig. 4 shows the geometry of the simple cavity E-plane metal insert filter. We discretize the structure in rectangular cells 0.762 mm × 0.75 mm wide using a time step of 1.25 ps. The stability factor is 0.87 and the number of iterations is 30 000. The excitation function is a standard pulse modulated by a sinusoid. The ratio minimum between the wavelength in the free space and cell dimension is larger than 30. Computer time and memory occupation are insignificant. Figs. 5 and 6 show the magnitude and the phase of the S_{11} parameter using now a simple first-order Mur's ABC without and with correction. The corrected results agree with those previously obtained.

IV. Conclusion

We introduce a simple and fast technique to improve the evaluation of scattering parameters using FDTD. This tech-

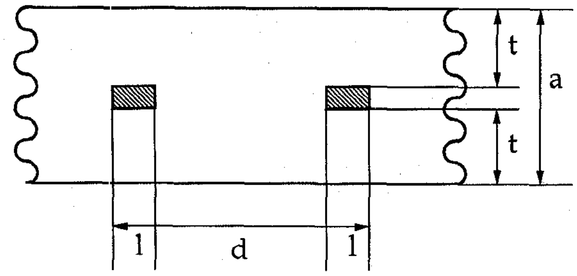

Fig. 4. Top view of the proposed test geometry: it is a simple cavity metal insert E-plane filter. Waveguide dimensions are $a = 22.86$ mm and $b = 10.16$ mm; $t = 10.67$ mm, $l = 3$ mm; $d = 21$ mm.

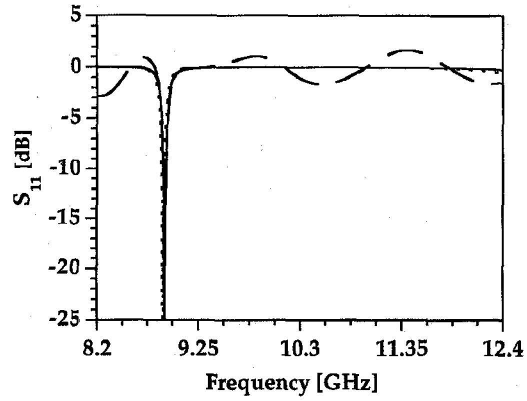

Fig. 5. Magnitude of the S_{11} parameter using first-order Mur's ABC without correction (- - - -) and with correction (———). Dotted line refers to variational analysis.

nique corrects residual reflection in the frequency domain subsequently to field evaluation in the time-domain. Hence, it is independent of the kind of line and applicable to microstrip, classical waveguide or any other transmission media. The technique can also be applied to any kind of ABC.

We presented formulae and results for a two-port symmetric structure, but the method can be easily extended to the case of an asymmetric multiport by performing a short, supplementary

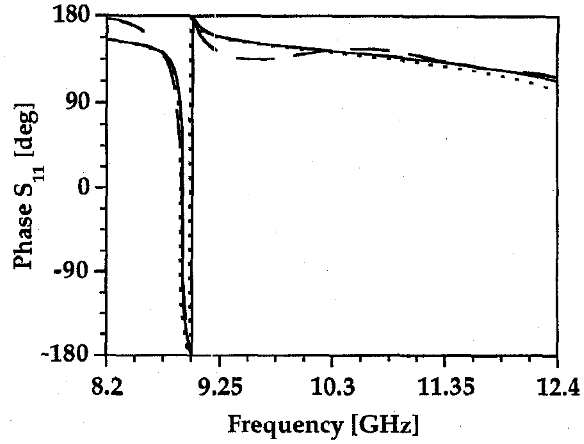

Fig. 6. Phase of the S_{11} parameter using first-order Mur's ABC without correction (- - -) and with correction (———). Dotted line refers to variational analysis.

initial run in order to evaluate the termination of each line involved.

At the cost of this short additional run, the proposed technique produces very accurate scattering parameters in any guiding environment with any ABC.

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